The effect of inclination on the integrated flux is shown in Fig 3 In this figure, the ratio of equivalent time for a particular inclination to the equivalent time for an equatorial spiral is plotted vs inclination A considerable gain can be realized by increasing the inclination a small amount in the range of low inclinations For a given acceleration, the integrated flux can be reduced by more than a factor of four by selection of a polar spiral rather than an equatorial spiral for a particular mission

For accelerations not exceeding 10^{-2} m/sec^2 (above which the starting point of the trajectory becomes an important parameter), the data of Figs 2 and 3 can be expressed by the simple relation, equivalent time (hr) $= K_1K_2/a$, where a is acceleration, m/sec^2; K_1 is 0 25 hr-m/sec^2, a constant obtained fiom Fig 2; and K_2 is the ratio of equivalent time for inclination I to equivalent time for inclination of 0°, obtained from Fig 3

As an example of the significance of the presented results, Fig 2 indicates that a spacecraft traveling with an acceleration of 5×10^{-3} m/sec^2 in a plane inclined 28 4° would accumulate a dose equivalent to a residence of 28 5 hr in the heart of the proton belt With the representative shielding data of Ref 5, this would correspond to a required shield density of 100 g/cm^2 of aluminum if the dose were to be limited to 30 rem from this source

References

[1] Barnes, T G , Finkelman, E M , and Barazotti, A L , "Radiation shielding of lunar spacecraft," *Lunar Exploration and Spacecraft Systems* (Plenum Press, Inc , New York, 1962) pp 52–71

[2] Perry, F C , ' Proton fluxes along trajectories through the inner Van Allen belt," *Protection Against Radiation Hazards in Space, Book II: Proceeding of the Symposium,* Atomic Energy Commission TID 7652 (1962), pp 725–738

[3] Allen, R I , Dressler, A J , Perkins, J F , and Price, H C , ' Shielding problems in manned space vehicles, ' Lockheed Nuclear Products Rept 104 (1960)

[4] Moeckel, W E , "Trajectories with constant tangential thrust in central gravitational fields," NASA TR R-53 (1960)

[5] Beck, A J and Divita, E L , "Evaluation of space radiation doses received within a typical spacecraft," ARS J **32**, 1668–1676 (1962)

Simplified Calculation of the Jet-Damping Effects

NICHOLAS ROTT*
University of California, Los Angeles, Calif
AND
LEMBIT POTTSEPP†
Douglas Aircraft Company, Inc , Santa Monica, Calif

IT has become traditional, since the work of Rosser, Newton, and Gross,[1] to calculate the jet damping of rockets as a sum of two effects: 1) the angular momentum flux carried by the exhaust, and 2) the effect of change of the moment of inertia Thomson[2] calculated the effect for a cluster of jets based on the same definitions

Fundamentally, the calculation of the jet damping involves the division of the rocket mass into two systems, namely, the rigid frame and the fuel moving relative to the frame The jet-damping effect can be defined as that part of the reaction of the fuel (i e , force and moment) on the rigid frame which arises from the angular velocity of the rocket in pitch or yaw In this note, the total effect will be calculated in one step by a method that leads to some simplifications, both conceptually and practically, and is applicable to both liquid and solid propellant rockets

The reaction of the propellant on the rocket will be determined from the equations of motion for the propellant in the fluid phase, which will be written down presently in a rotating frame For the purposes of jet-damping calculations, the fluid flow will be assumed inviscid, although not necessarily irrotational This is permissible as viscosity affects mainly the "basic" velocity distribution of the fuel flow in a nonrotating rocket, which can be approximated closely by an inviscid flow with proper vorticity distribution Thus, the effect of viscous stresses will be neglected, but the viscous effects on the basic flow distribution can be accounted for

A further simplification, which is also implied in all previ ous calculations, is the "quasi-steady" approach to be used presently By definition, jet-damping effects include reac tions due to angular velocity and not those arising from angular accelerations To exclude the latter effect, only the limit of very small accelerations will be considered, and the corresponding terms in the equations of motion will be omitted The angular velocity will be taken as a parameter varying so slowly that the calculations can be made "as if" the rotation were steady

No simplifying assumptions will be needed concerning the compressibility of the fluid It is also permissible that the total energy and the entropy vary from streamline to streamline

Let the equations of motion be written down in a rocket fixed Cartesian system (Fig 1) with the x axis parallel to the thrust line To fix ideas, only one component ω_z of the angular velocity of the rocket will be taken as nonzero; this means that the effect of roll (ω_x) is not the subject of the present investigation The equations of motion are as follows:

$$\left.\begin{aligned} u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} - 2\omega v &= -\frac{1}{\rho}\frac{\partial p}{\partial x} + \omega^2 x \\ u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + 2\omega u &= -\frac{1}{\rho}\frac{\partial p}{\partial y} + \omega_z^2 y \\ u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} &= -\frac{1}{\rho}\frac{\partial p}{\partial z} \end{aligned}\right\} \qquad (1)$$

The aim is to calculate the effect of a small angular velocity ω on the pressure field p Accordingly, the effect of the "centrifugal" terms containing ω_z^2 will be neglected in comparison to the Coriolis terms proportional to ω

An elementary calculation, which provides all the essential results, can be made after making an apparently radical simplifying assumption on the flow field Let it be assumed that the fuel motion in the rocket is everywhere parallel to the x axis The fuel velocity u and the density ρ may, however,

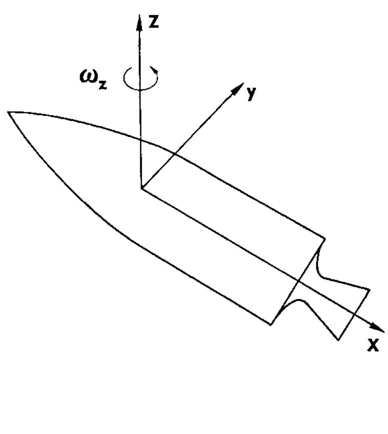


Fig 1 Coordinate system

Received January 15, 1964

* Professor of Engineering; also Consultant, Douglas Aircraft Company, Inc , Santa Monica, Calif Associate Fellow Member AIAA

† Chief, Flight Mechanics Research Section, Advance Missile Technology Member AIAA

vary over any cross section of the duct or nozzle in an arbitrary way For such a "hydraulic" theory, the only relevant equation is

$$2\omega\, u = -\frac{1}{\rho}\frac{\partial p}{\partial y} \tag{2}$$

Integration in y direction in a given cross section gives (see Fig 2)

$$p_2 - p_1 = -2\omega \int_{y_1}^{y_2} \rho u dy \tag{3}$$

and the force in y direction per unit length of the rocket becomes

$$F_y = \int_C (p_2 - p_1)dz$$
$$= -2\omega\, \dot{m} \tag{4}$$

where $\dot{m}$ is the total mass flux per unit time in the cross section under consideration

From the force per unit length, the total force expressing the reaction of the moving fuel on the rocket due to the Coriolis effect is easily determined; in particular, for a liquid fuel system, where $\dot{m}$ = const, the total force is

$$R_y = -2\omega\, \dot{m}(x - x_0) \tag{5}$$

where x_0 and x are the coordinates of the liquid-free surface in the tank and of the exit area, respectively The jet-damping moment (with respect to the point $x = 0$) is then

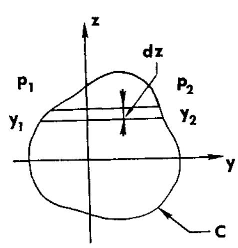
$$N = -\omega\, \dot{m}(x^2 - x_0^2) \tag{6}$$

The factor $\dot{m}x_0^2$ may be interpreted, if desired, as the change of the moment of inertia per unit time in "hydraulic" approximation This is more accurate than the estimate used by Rosser et al ,[1] which explains the small difference in the final results Besides, the traditional subdivision of the jet-damping effect into the angular momentum flux and the change of the moment of inertia appears rather artificial from the point of view of the calculations presented here, as it is easier to calculate the "total" directly than the parts

Reviewing the effect of the simplifying assumptions made previously, first it should be noted again that only the mass flux $\dot{m}$ enters the result expressed by Eq (4) Thus, the distributions of the velocity and the density in any cross section leave the result unaffected, even if they are modified by the angular velocity The only effect that would be of interest is the influence of ω on $\dot{m}$ Formally, this is a second-order effect, and practically, it can be expected to be very small

To estimate the actual effect of rotation on the velocity profile, the Helmholtz-Kelvin vorticity conservation laws for frictionless fluid (which has been assumed) can be formulated in an inertial frame of reference in which the rocket has slowly changing angular velocity If use is made of the hydraulic approximation, it can be deduced that the change in velocity profile will consist of a linear distribution with the gradient ω in y direction (The situation is quite analogous to the determination of the so-called "slip" in a radial compressor, which can be found in the literature [3]) As ω_z only fixes the value of the velocity gradient, the influence on $\dot{m}$ (if any) cannot be obtained from such considerations, but

Fig 2 Force evaluation in an arbitiary duct cross section



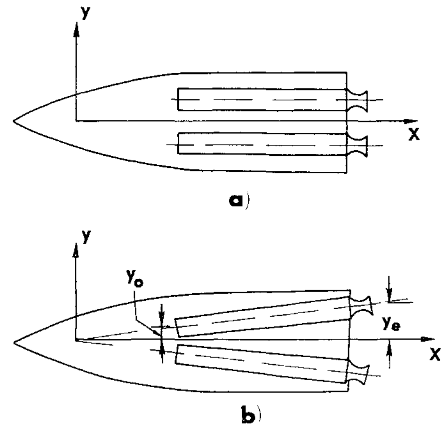


Fig 3 Rocket cluster arıangements

$\dot{m}$ = const is clearly compatible with this main effect of the rotation on the flow profile

The simplification that is most significant to the degree of approximation considered here is the hydraulic assumption made previously A simple estimate of the error to be expected can be obtained by considering problems of rocket clusters

All significant properties of clusters can be observed in the case of two rocket motors in the x-y plane Two cases will be compared as sketched in Fig 3 In case 3a, the two motors are arranged parallel to the x axis The jet damping moment of each engine, if calculated by the principles used previously, will not be influenced by the parallel offset of the thrust axis against the plane of symmetry; Eq (6) remains unchanged if $\dot{m}$ for the total flux of both rockets is inserted

The situation is different in the case represented by Fig 3b The two motors form an angle; let it be assumed that their thrust lines intersect at the point $x = 0$ with respect to which the jet damping moment is to be calculated Then the jet-damping moment of each engine still can be found by use of Eq (6), after a change of coordinates For each engine, the x axis must be turned such that it coincides with the thrust line The sum of both moments is found, expressed in the original coordinate system of Fig 3b, to be

$$N = -\omega\, \dot{m}[(x^2 - x_0^2) + (y^2 - y_0^2)] \tag{7}$$

where $\dot{m}$ again is the total flux for both rockets

It is seen that, if the hydraulic assumption would have been made for the cluster shown in Fig 3b (i e , if the y component of the velocity in the original coordinate system would have been neglected), the term $y^2 - y_0^2$ in Eq (7) would be missing

If a single rocket engine is considered as a cluster of stream-tubes, the previous result indicates that the error caused by the hydraulic assumption is of the order of the square of the lateral dimension (i e , the cross-sectional area) to the square of the longitudinal dimension Preliminary investigation has shown that, if the hydraulic assumption is to be abandoned, the problem becomes much more complicated Cases where the error just estimated is too large may occur, so that an improvement of the present theory in this direction is desirable It may be added that the error inherent in all previous theories is of the same order

Returning to possible new uses of the simple theory presented here, let us consider the case of solid-fuel rocket engines It is proposed to make use of the fundamental Eq (4) but to let $\dot{m}$ be replaced by a variable flux $\dot{\mu}$ growing according to the production of combustion gases in the motor A simple illustrative example is obtained assuming that $\dot{\mu}$ grows linearly between x_0 and x_n, where $x_n(<x)$ is the coordinate of the entrance to the nozzle Thus,

$$\dot{\mu} = [(x - x_0)/(x_n - x_0)]\dot{m}_t \tag{8}$$

where $\dot{m}_t$ is the total flux ($\dot{\mu} = \dot{m}_t$ between x_n and x) The jet-damping force is now

$$R_y = -2\omega \dot{m}_t \left\{ \int_{x_0}^{x_n} \frac{x - x_0}{x_n - x_0} dx + (x - x_n) \right\}$$
$$= -2\omega \dot{m}_t [x - \tfrac{1}{2}(x_n + x_0)] \quad (9)$$

For the jet-damping moment a similar calculation yields

$$N = -\omega \dot{m}_t [x^2 - \tfrac{1}{3}(x_n^2 + x_n x_0 + x_0^2)] \quad (10)$$

If the result is compared with Eq (6) in the special case $x_n = x_e$, $x_0 = 0$, the value given by Eq (10) is different by a factor $\frac{2}{3}$, which is not negligible

References

[1] Rosser, J B , Newton, R R , and Gross, G L , *Mathematical Theory of Rocket Flight* (McGraw Hill Book Co , Inc , New York, 1947), Chap 1, pp 19–23

[2] Thomson, W T , *Introduction to Space Dynamics* (John Wiley and Sons, Inc , New York, 1961) Chap 7, pp 221–226

[3] Smith, C W , *Aircraft Gas Turbines* (John Wiley and Sons, Inc , New York, 1956), Chap 10, pp 206–207

Vortex Flow in Arc Heaters

K E Tempelmeyer* and L E Rittenhouse†
ARO, Inc , Arnold Air Force Station, Tenn

Introduction

As a result of the increasing use of electric arc heaters, both to generate high-enthalpy streams for testing purposes and for space propulsion, various investigators (e g , Refs 1–4) have attempted to analyze the detailed mechanism of the energy exchange between the electric arc and the gas stream In such analyses the arc column geometry is postulated, and various flow regimes are considered separately Despite its simplifications, the Stine-Watson theory,[2] for example, appears to provide good results for the so-called constricted-arc configuration, but in it, as well as in most other theoretical analyses, the flow is considered to be one-dimensional However, in many arc-heater designs, vortex flow is used to induce arc rotation and thereby minimize electrode erosion The purpose of this note is to summarize some effects of vortex flow on the operation of Gerdien-type and constricted-arc configurations and, as a result, attempt to indicate the validity of applying irrotational energy exchange theories to vortex-flow or gas-stabilized arc heaters

Discussion

A typical Gerdien arc-heater geometry is shown in Fig 1a and the very similar constricted-arc configuration in Fig 1b Since much of the energy addition occurs in a constant-area cylindrical passage for these configurations, it appears reasonable, as noted in Refs 5 and 6, to approximate the heating process gas dynamically by Rayleigh flow

Rayleigh-type heat addition may accelerate subsonic flows to sonic velocity If, however, the amount of heat transferred to the gas is greater than that required to produce sonic velocity at the exit of the constant-area heating section,

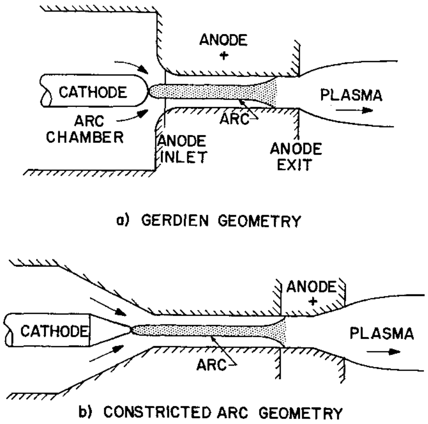


Fig 1 Arc-heater configuration

which would normally be the case in an arc heater, some adjustment of the entry conditions must occur Before striking an arc the cold gas is accelerated, nearly isentropically, from the arc chamber stagnation pressure and temperature to the nozzle exit static pressure and temperature The cold flow can already be sonic at the exit of the constant-area nozzle passage and nearly sonic at the entrance Consequently, the addition of heat to the gas, which begins when the arc is struck, must cause some readjustment of the flow conditions entering the constant-area passage As pointed out by Shapiro,[7] this adjustment is brought about by a series of transient pressure waves which propagate in both directions along the channel, changing either or both the mass flow and pressure level until the velocity or Mach number at the entrance of the constant-area section decreases to allow the required amount of heating to occur with sonic velocity at the exit

From a Rayleigh analysis one would conclude that arc heating rather than gasdynamic expansion is primarily responsible for the acceleration of the plasma to $M = 1\ 0$ at the exit of the constant-area section This effect is clearly demonstrated by the anode pressure distributions shown in Fig 2, which were obtained with a Gerdien configuration (see Ref 6 for details) Without an electric arc, the cold flow enters the anode passage at a Mach number of about 0 7 and is accelerated to sonic velocity at the exit by frictional effects The pressure distribution demonstrates that, when the arc is initiated, the flow readjusts itself to a very low anode entry Mach number ($M \ll 0\ 1$) and is accelerated rather uniformly by the heat addition When the gas is introduced tangentially into the arc heater to produce a vortex flow, the resulting anode pressure distributions follow the same pattern Since the heat-addition process greatly decreases the axial velocity component at the entrance of the anode passage, the velocity in the arc-heater stilling chamber is greatly decreased Consequently, it seems reasonable that the tangential velocity component for the vortex-flow case would also be significantly decreased because of the longer "dwell time" and increased viscous dissipation in the stilling chamber Also, when the mass flow decreases as a result of heat addition, the tangential injection velocity decreases Thus, it might be expected that the arc heat addition through either of the mechanisms would tend to weaken the vortex or any other flow nonuniformities arising in the arc-heater stilling chamber

Some impact pressure surveys, which illustrate this point, have been made in free jets from arc heaters of the geometry shown in Fig 1a Pressure profiles taken at two nozzle diameters from the exit plane of a sonic jet are shown in Fig 3 for vortex and nonvortex flows (see Ref 6 for details) For the vortex case, argon was injected tangentially into the arc chamber at a velocity of about 100 fps From angular momentum considerations a tangential velocity of about 500 fps would be expected at the wall of the anode passage for

Received January 31, 1964 The research material reported in this paper was sponsored by the Arnold Engineering Development Center, U S Air Force Systems Command, under Contract No AF 40(600)-1000 with ARO, Inc Further reproduction is authorized to satisfy needs of the U S Government

* Supervisor, MHD Section, Propulsion Wind Tunnel Facility Member AIAA

† Research Engineer, MHD Section, Propulsion Wind Tunnel Facility Member AIAA